# Quasi-Static Moment Measurements for Airfoils in an Annular Cascade

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Typical aerodynamic moment measurements are presented for airfoils in an annular cascade executing very slow pitching oscillations. Parameters which are varied are mean incidence, interblade phase angle, and amplitude of oscillation. The measurements take the form of a continuous loop of moment vs angular displacement. The characteristics of the experimental data are discussed and the need is pointed out for an analytical technique for converting the quasi-static data to dynamic moment loops for application to flutter prediction.

#### Nomenclature

 $a_n$ ,  $b_n$  = Fourier coefficients

b = airfoil semichord

 $= \text{moment coefficient, } C_M = M/(2\rho W_1^2 b^2)$ 

 $k = \text{reduced frequency}, k = \omega b/W_1$ 

M = aerodynamic moment

t = time

 $W_1$  = relative approach velocity

 $\alpha_1$  = mean incidence measured from  $W_1$  to chordline

 $\sigma$  = interblade phase angle

 $\rho$  = air density

 $\omega$  = oscillation frequency

 $\theta$  = angular displacement of airfoil

 $\theta_0$  = amplitude of angular displacement

s/c = space/chord ratio

 $\beta$  = stagger angle

# Introduction

T is now firmly established on both theoretical and experimental grounds that the flutter problems encountered in conventional subsonic axial-flow turbomachines are associated with flow separation. Some controversy¹ remains as to whether destructive blade vibrations may be ascribed to such stalling flutter or whether the phenomenon is in fact a forced resonance with propagating or rotating stall cells. However, in recent years it has become clear that stall flutter in a predominantly torsional mode is a significant hazard which must therefore be understood and guarded against.

Since a great deal of importance attaches to oscillatory torsional blade motion, particularly in fans and compressors, a like amount of importance must be attributed to the aero-dynamic moment experienced by a cascaded airfoil executing such motion. This information, of a time-dependent nature, is necessary to predict torsional flutter and would also play a central role in any torsional resonance mechanism.

Under so-called "classical" subsonic conditions of small foil incidence, camber and thickness the time-dependent variation of moment with position during a prescribed harmonic torsional oscillation is well-predicted by a number of theoretical, two-dimensional treatments.<sup>2-4</sup> In particular Nemesh<sup>3</sup> treats the blade thickness near the leading edge in a significantly improved manner according to singular perturba-

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tion theory. Except for applications involving forced vibrations under conditions of unseparated flow these results are not of immediate practical utility. They are of some significance, however, in providing accurate limiting behavior at the low end as the mean incidence is varied from small values on up to high (positive or negative) values associated with separation of flow and stall of the airfoil.

A purely analytical procedure for the evaluation of nonsteady stalling behavior seems to be beyond the capability of a necessarily inclusive yet tractable theory. Prediction of static flow separation on a single airfoil is questionable in itself without adding requirements for detailed pressure distribution under dynamic conditions and in cascades. For this reason it has seemed most propitious to seek a predictive method within the framework of a semiempirical technique. 5,6 By measuring the values of aerodynamic moment, for example, under quasi-static conditions a large number of theoretical questions are by-passed; the experimental apparatus becomes in essence an analog computer accounting for many of the boundary layer and separation effects that are operative. The problem which remains to be solved analytically is the conversion of these measured quasistatic moment characteristics to full dynamic characteristics which may then be used in stall flutter analysis or other aerolastic investigation.

## **Description of Research Programs**

The over-all objective of the program described herein is to develop the capability of predicting the time-dependent variation of aerodynamic moment given the variation of torsional displacement with time. The motion is assumed to be harmonic. If the moment, or preferably moment coefficient, is plotted against angular position the time assumes the role of a parameter and the resulting curve is conventionally termed a moment loop. Hence the entire procedure may be called moment loop prediction. Presupposing that the blades in the cascade are all executing the same motion with a constant phase angle between the motion of adjacent blades, then the governing physical variables are mean incidence  $\alpha_1$ , vibration amplitude  $\theta_0$ , reduced frequency of vibration k, Reynolds number, Mach number, interblade phase angle  $\sigma$ , chordwise position of the axis of rotation and a complete definition of cascade geometry (stagger, solidity and profile shape).

The research program is divided into four major phases. Phase 1: design, construction and calibration of an annular cascade wind tunnel; phase 2: procurement of experimental moment coefficient data at reduced frequencies approaching zero (i.e., quasi-statically); phase 3: development and application of an analytical technique for supplying the finite frequency effect; and phase 4: prediction of flutter behavior of a particular system and corroboration of the overall method by conducting free flutter tests of the same configuration. The present paper is concerned chiefly with the first two phases.

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## Wind-Cage Tunnel Design

The Stevens annular cascade tunnel, shown in Fig. 1, has a vertical axis with flow entry from the top. The entrance section consists of a conventional bellmouth and nose bullet. The main section of the tunnel is composed of a number of interchangeable cast aluminum rings, 6.00 in. in axial extent and machined to an inside diameter of 20.00 in. A similar set of rings finished to an outside diameter of 16.00 in. form the inner flow annulus wall. The two sets of rings are stacked vertically to provide sections of variable inlet guide vanes, instrumentation section, test section, and exit guide vane(s) section(s). The aluminum rings rest on the inlet flange of a large ventilation blower which induces flow through the tunnel and discharges downward onto the laboratory floor.

A number of sets of airfoils have been cast in epoxy with a single spanwise cylindrical steel shaft encastred in one end of each airfoil. The platform of each constant section foil is nominally a two inch square. A typical guide vane section will consist of thirty identical airfoils with shafts disposed radially and penetrating the outer ring through, in this case, thirty equispaced holes. The guide vane shafts are connected through individual bell cranks to a unison ring which allows

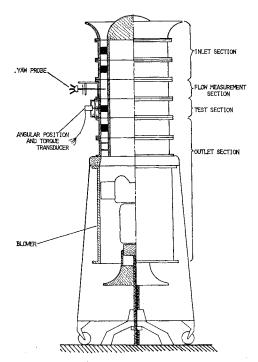


Fig. 1 Schematic of the experimental apparatus.

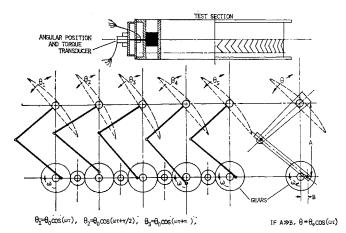


Fig. 2 Linkage and gear train arrangement for sinusoidal motion with 90° interblade phase angle.

rapid changes of guide vane setting angle. Currently two sets of guide vanes, one with  $15^{\circ}$  camber and the other with  $40^{\circ}$  camber may be used singly or in series to produce different amounts of swirl entering the instrumentation and test sections

The instrumentation section which follows the guides is used for initial calibration and surveys of the flowfield entering the test section. Following these initial tests the yawmeter and pitot-static tube are used to read flow angle and magnitude in subsequent data taking runs.

The test section used for the Phase 2 experiments consists of either 20, 30, or 60 identical blades each with its shaft terminated in a small four-bar link mechanism. This mechanism, designed to closely generate a harmonic motion of the shaft, appears in Fig. 2. A number of small nylon gears, with idlers interposed to preserve correctly the directional characteristics of shaft motion, form the drive into the linkage from a small pin set eccentrically to the gear centerline. By placing the pin in one of two positions a large (6°) or small (2.6°) blade oscillatory amplitude may be selected.

The entire gear train is driven simultaneously at four peripheral locations by small, low speed geared a.c. motors. Because of the low output speed of the motors (2 rpm) the reduced frequency of the ensuing motion is virtually zero for any finite airspeed.

Interblade phase angle may be changed by removing the idler gears, indexing the remaining gears relative to each other and then replacing the idlers.

The shafts of four selected airfoils are fitted with extremely sensitive strain gage torque transducers such that the small elastic torsional deflection of the shaft necessary to measure torque are negligible compared to the gross torsional displacement of the entire shaft and airfoil system. The strain guage signal, suitably bridged, is fed to one axis of an x-y recorder. The other signal is supplied by a linear variable differential transformer linked to measure torsional displacement. A moment loop is therefore produced directly by the x-y recorder output; suitable calibrations and a simultaneous airspeed measurement serve to quantify the raw data.

A row of exit guide vanes operate to partly or wholly deswirl the flow leaving the test section and entering the blower.

Flow velocity in the annulus may be varied by discharge throttling of the blower (which runs at constant speed) or by ventilating the nose bullet to bypass some inlet flow directly through the central cylindrical void into the blower suction.

## **Typical Moment Coefficient Data**

Owing to the complete dearth of experimental moment data on cascaded airfoils the quasistatic results are of fundamental interest from a purely qualitative standpoint. Setting the

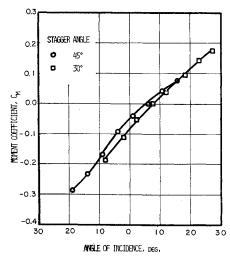
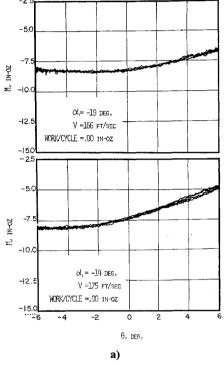
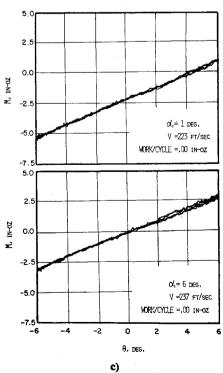


Fig. 3 Moment coefficient curves.

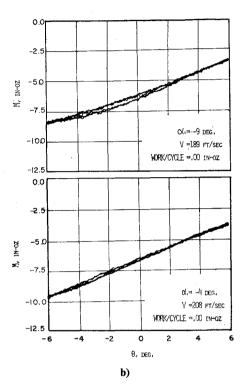
interblade phase angle at zero allows the static moment coefficient characteristic to be traversed, see Fig. 3. Very generally the stall of a cascaded airfoil is seen to be much less precipitous than the typical isolated airfoil. Evidently the channeling effect of neighboring airfoils serves to prevent the sudden formation of a large separated area on the suction side of the airfoil for a small change in incidence. Although stall does occur as evidenced by a nonlinear variation of moment with incidence, the nonlinearity is not as strong, or potentially discontinuous, as it is for an isolated airfoil of comparable small thickness.<sup>7</sup> This fact is apparently of some importance in subsequent phases of the program which have as their objective the prediction of finite frequency effects.





In Figs. 4 through 8 are displayed a large number of moment loops for a cascade of double circular arc airfoils with 45° stagger and 0.842 pitch/chord ratio. The airfoils have approximately 15° camber and 7% thickness. The first four figures are for a "vibration" half amplitude of 6° while the last figure is for a smaller amplitude of 2.6°. Each figure shows 8 separate loops for incidences varying from negative stall through zero to positive stall.

In Fig. 4, the case of zero interblade phase angle has been recorded. The loops are seen to collapse into a single line emphasizing the fact that the configuration of the blades in the cascade are identical for a given angular displacement irrespective of whether the displacement is increasing or decreasing



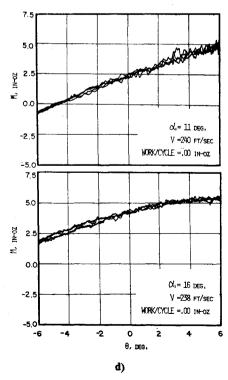


Fig. 4 Moment loops for large amplitude with 0° interblade phase angle, 45° stagger angle and 0.84 space/chord ratio.

with time. The single line characteristic is a further result of the quasistatic nature of the experiments; no aerodynamic lag is present or possible unless generated by a hysteresis effect attributable to a bistable flow pattern. In practice it is possible to detect very thin loops stemming from slight inaccuracies in setting adjacent blade angles and from a small residual error in the motion; the four bar linkage output is not quite a pure sinusoid.

Another typical feature of all the loops is the high frequency random vibration superimposed on the generally smooth loop contours. The "noise" is attributable to the turbulence in the airstream which is sometimes enhanced when the test blade is immersed in the wake of a guide vane. Although this wake effect does not appear to invalidate the data, in future tests out-of-wake data will be obtained for comparison. The general turbulence level is thought to be fairly representative of actual turbomachine practice but has not been measured. The relatively large response is associated with the blade mounting shaft compliance which is not representative of an actual turbomachine blade.

Figure 5 and 6 show the effect of finite interblade phase angles of 60° and 120°, respectively. Here is observed not only the important loop-producing function of phasing, but also the very complicated and crucial loop crossing attributable

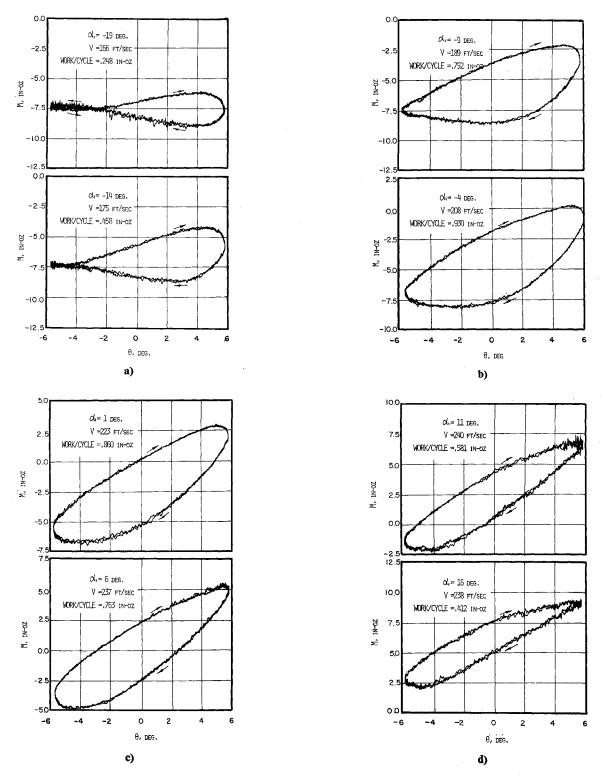


Fig. 5 Moment loops for large amplitude with 60° interblade phase angle, 45° Stagger angle and 0.84 space chord ratio.

to stalling of the test airfoil. The direction of traversal of the loops is shown by arrows.

It should be emphasized that the loops are produced by interblade phase angle and not aerodynamic lag associated with finite frequency of oscillation. In fact the descriptor "quasi-static" was chosen in favor of "static" to indicate that although the test frequencies are vanishingly small (k approximately 0.0015), the vestiges of time dependence remain in the need to specify direction of loop traversal. Thus phase angles of  $60^{\circ}$  and  $300^{\circ}$  produce the same quasi-static loops but they are traversed in opposite directions. One may note also at this point the need for a dynamic loop prediction technique

to accept a quasi-static loop as an input whereas for an isolated airfoil a single line characteristic suffices.

Generally phase angles  $0 < \sigma < 180^{\circ}$  can be shown to be destabilizing. Thus for high enough airspeed a low enough reduced frequency is produced such that the cascade representing a row of compressor blades will flutter with interblade phase angles in the aforementioned range. These considerations are based on the fact that the work done on the oscillation by the airstream is given by work/cycle =  $\oint Md\theta$  and represents the (signed) area of the moment loop. The previous conclusion may be modified by a consideration of vibration at finite reduced frequency an/or loop crossing

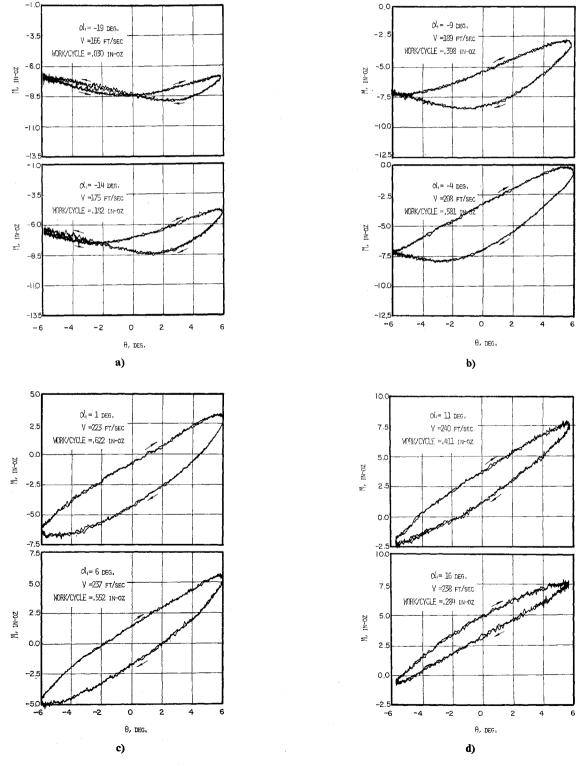


Fig. 6 Moment loops for large amplitude with 120° interblade phase angle, 45° stagger angle and 0.84 space/chord ratio.

attributable to stalling. At or near stall conditions the complicated interaction of the quasi-static moment characteristic and the nonstationary aerodynamic modifications which produce the dynamic loop permit work to be fed into the motion at much higher reduced frequencies. When loop crossing begins to occur the signed area of the loop begins to decrease algebraically and may eventually change sign.

Figure 7 shows the behavior of the quasi-static moment loop when antiphasing is enforced ( $\sigma = 180^{\circ}$ ). Here again a single line characteristic is obtained emphasizing that with this particular phasing the configuration of the blades in the

cascade is independent of whether the angular displacement is increasing or decreasing. Flutter with antiphased motion is not prohibited since at finite frequencies a true loop may be obtained; in fact, many of the earlier investigators of cascade flutter reported phase angles of approximately 180° (Refs. 9 and 10).

Figure 8 shows moment loops with the smaller amplitude of torsional motion. It is seen to be more unusual to detect crossing of the loops. This observation is consistent with the limiting behavior that one would expect for motions of extremely small amplitude; an elliptical loop is traced out

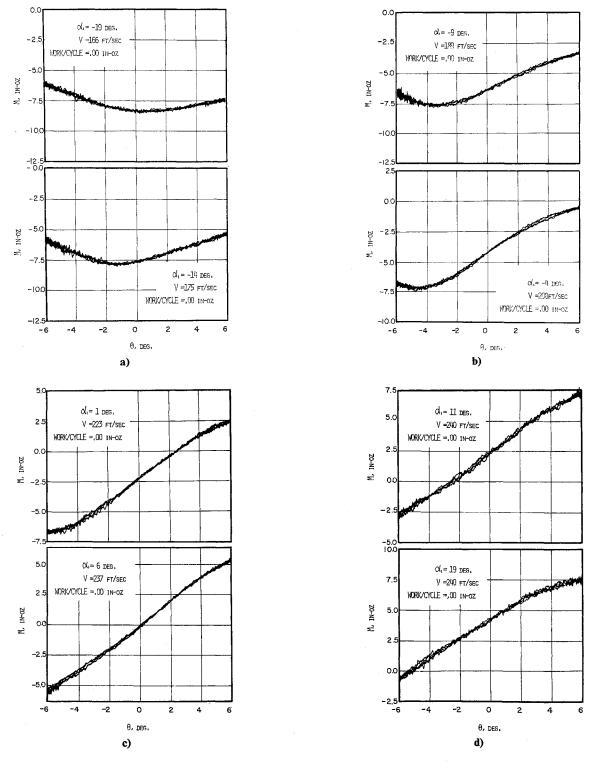


Fig. 7 Moment loops for large amplitude with 180° interblade phase angle, 45° stagger angle and 0.84 space/chord ratio.

about a center on the static moment characteristic such as Fig. 3. For special phasings of zero and 180° these small ellipses would collapse into straight lines, though not necessarily tangent to the static curve.

The limiting behavior emphasizes an important feature of the energy transfer. A destabilizing loop area for an infinitesimal loop may increase with increasing torsional amplitude, but ultimately the area will decrease and approach zero for sufficiently large amplitude. Thus, in principle, it becomes possible to predict an equilibrium amplitude of vibration. The dynamic moment loops must be used to be correct, but

the qualitative argument can be based on the static and quasistatic considerations described previously.

#### **Reduced Date**

From a number of runs in which the quasistatic loop areas have been planimetered it is possible to observe the effect of interblade phase angle on work per cycle. This has been plotted in Fig. 9 for a particular cascade geometry and flow angle. The critical nature of interblade phase angles in the neighborhood of 60° has been noted by other investigators.<sup>11</sup>

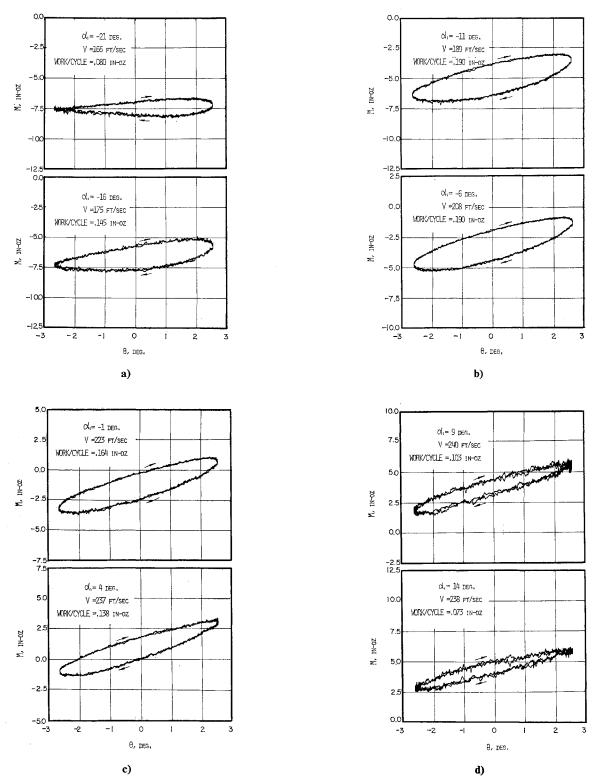


Fig. 8 Moment loops for small amplitude with 60° interblade phase angle, 47° stagger angle and 0.84 space/chord ratio.

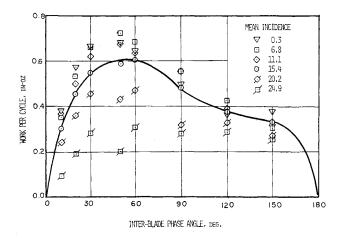


Fig. 9 Work per cycle vs interblade phase angle under various mean incidence for 6° amplitude with 25° stagger angle and 0.42 space/ chord ratio.

By performing a harmonic analysis with the displacement given by  $\theta = \theta_0 \cos \omega t$  and the moment coefficient by

$$C_{M} = a_{0} + \sum_{1}^{\infty} a_{n} \cos n\omega t + \sum_{1}^{\infty} b_{n} \sin n\omega t$$

it may be shown that the work done on the vibration is proportional to the single coefficient  $b_1$ ; Work/cycle =  $-2\pi\rho W_1^2b^2\theta_0b_1$ . The collection, correlation and dissemination of moment data therefore reduces to the recording of values of  $a_1$  and  $b_1$  for quasi-static loops as a function of the variables on which they depend. Any successful dynamic loop prediction method developed in Phase 3 of the program will calculate an equivalent  $b_1$  for the dynamic loop which may then be used directly in stall flutter prediction.

#### Conclusions

- 1) Quasi-steady considerations alone explain the possibility of flutter when interblade phase angle is a free parameter. The lack of the appearance of flutter in these cases must be attributed to the presence of material and structural damping and to the aerodynamic components of damping that appear under dynamic (finite frequency) conditions.
- 2) The importance of either positive or negative stall should not be overlooked. For typical subsonic and transonic blade sections the leading edge radius and nose camber cooperate to produce a more sudden stall in the negative incidence regime. For a cascaded airfoil with a narrow working region of incidence an appreciable torsional amplitude

- of vibration may permit the dynamic penetration of the stall regime much more easily than might at first be supposed. The significance of the static characteristics is not overly great in making this determination since the existence of an interblade phase angle other than zero complicates and obfuscates the meaning of the conventional plot of moment vs incidence.
- 3) In general an airfoil in cascade stalls more gently than its isolated counterpart. The greater the pitch/chord ratio. the less this may be expected to be true.
- 4) The need for experimentally recording the quasi-static moment coefficient variation, particularly near stall, has been demonstrated and the method for doing this had been put on a rational basis. Some sample data shows the trends which may be expected with mean incidence, interblade phase angle and oscillation amplitude. A routine cataloging of many parametric variations remains to be done.
- 5) A key link in the over-all procedure of successful dynamic moment loops prediction remains to be perfected. This link is the analytical conversion of quasistatic to dynamic characteristic; i.e., the addition of a valid finite frequency effect.

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